Residual Stresses in a Laminated Shell During Cure

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In this paper, a viscoelastic finite element analysis was performed to investigate residual stresses occurred in a laminated shell during cure. A viscoelastic constitutive equation that can describe stress relaxation during the cure was defined as functions of degree of cure and temperature, and derived as a recursive formula used conveniently for numerical analyses. The finite element program was developed on the basis of 3-D degenerated shell element and the first order shear deformation theory, and was verified by comparing with an exact solution of the one dimensional problem. Effects of chemical shrinkage and stacking sequence on the residual stresses in the laminated shell during the cure were investigated. The results showed that there were big differences between viscoelastic stresses and linear elastic stresses calculated by considering thermal deformation and the chemical shrinkage induced by the degree of cure.

Key Words : Viscoelastic Residual Stress, Cure, Degree of Cure, Chemical Shrinkage, Laminated Shell, Finite Element Analysis

1. Introduction

A residual stress occurred in fiber-reinforced composite materials during cure is one of severe factors that deteriorate performance of a composite structure. The residual stress can cause matrix cracking in composites before external loads are applied and thus can reduce the stiffness of the composites. In addition, since the residual stress means pre-loading, it can cause the degradation of strength. Therefore, the residual stresses induced during the cure should be considered in the design of the composite structure because they generally have bad effects on the stiffness and strength of the composite structure.

The first analysis of the residual stresses in a thermoset matrix composite during the cure was performed by Hahn and Pagano(1975). They calculated linear elastic residual stresses under the assumption that a laminate was in a stress-free state at cure temperature immediately before cool-

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down. Hahn(1976) predicted the linear elastic residual stresses in a laminated plate during the cure using a laminate theory and studied their effects on ply failure. Bogetti and Gillespie(1992) analyzed the cure-induced residual stresses for thick laminates using an incremental elastic laminated plate theory and investigated the effect of chemical shrinkage on the development of residual stresses. Hodges et al. (1989) presented an experimental approach to find optimal curing conditions for reducing the residual stresses of carbon fiber/epoxy composite. Kim and Hahn (1989) analyzed elastic residual stresses during the cure by monitoring warpage of unsymmetric cross-ply laminates and by relating the warpage to the residual stresses. White and Hahn(1991) developed a process model for the investigation of viscoelastic residual stress development in laminates during cure processing and validated the model by the intermittent cure of unsymmetric cross-ply laminates in which processing induced residual curvatures were measured. Kim(1996) introduced a viscoelastic constitutive equation depending on degree of cure and temperature by performing stress relaxation tests and investigated the residual stresses of Hercules AS4/3501-6

composite during the cure by analyzing two dimensional and axisymmetric problems.

In this study, using viscoelastic data provided by Kim(1996), the residual stresses occurred in a laminated shell during the cure will be investigated by deriving a viscoelastic constitutive equation and by using a finite element method. The viscoelastic finite element program will be developed on the basis of 3-D degenerated shell element and the first order shear deformation theory, and will be validated by comparing with an exact solution of the one dimensional problem. Effects of temperature, degree of cure, chemical shrinkage, and stacking sequence on the residual stresses induced in the laminated shell during the cure will be considered.

2. Viscoelastic Constitutive Equation

A linear viscoelastic constitutive equation for relaxation of residual stresses occurred during cure can be expressed by the following hereditary integral (Flaggs and Crossman, 1981 and Lin and Hwang, 1989):

$$\sigma_i = \int_0^t C_{ij}(a, T, t-\tau) \frac{\partial \bar{\varepsilon}_j}{\partial \tau} d\tau \qquad (1)$$

$$\bar{\varepsilon}_j = \varepsilon_j - \beta_j \Delta T - \eta_j \Delta a \qquad (2)$$

where σ and ε represent the stress and strain, and C, β, and η represent relaxation modulus, coefficients of thermal expansion, and coefficients of chemical shrinkage, respectively. ΔT and Δa indicate the changes in temperature and degree of cure. t denotes time and τ is a dummy variable for integration. If the material shows thermorheologically simple behavior at constant degree of cure a_0 and there is no initial strain at time $t = 0$, then Eq. (1) can be written in the integral form as

$$\sigma_i = \int_0^t C_{ij}(a, \xi^t - \xi^\tau) \frac{\partial \bar{\varepsilon}_j}{\partial \tau} d\tau \qquad (3)$$

where ξ represents a reduced time and is defined as follows (Kim, 1996).

$$\xi^t = \int_0^t \frac{ds}{a_T(a_0, T(s))},$$

$$\xi^\tau = \int_0^\tau \frac{ds}{a_T(a_0, T(s))} \qquad (4)$$

where a_T represents a shift factor expressed as functions of the degree of cure and the temperature. The time-dependent relaxation moduli C_{ij} in Eq. (3) are expressed in a finite exponential series of the form (Kim, 1996):

$$C_{ij}(a, \xi) = C_{ij}^\infty + C_{ij}^* \sum_{m=1}^N W_m \exp\left(-\frac{\xi(a)}{\tau_m(a)}\right) \qquad (5)$$

where C_{ij}^∞ and C_{ij}^u represent fully relaxed moduli and unrelaxed moduli, respectively and $C_{ij}^* = C_{ij}^u - C_{ij}^\infty$. τ_m and W_m are relaxation times and weighting factors at the given degree of cure. All parameters described above are determined from a viscoelastic experiment. Substituting Eq. (5) into Eq. (3), the viscoelastic constitutive equation is written as Eq. (6)

$$\sigma_i^t = \int_0^t \left[C_{ij}^\infty + C_{ij}^* \sum_{m=1}^N W_m \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \right] \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau \qquad (6)$$

We will now proceed to simplify the hereditary integral appearing in the constitutive equation in the manner proposed by Henriksen(1984) and Kennedy and Wang(1994) and derive a recursive formula that the solution at the current time t can be obtained from the solution known at the previous time $t - \Delta t$. First of all, Eq. (6) can be written as follows

$$\sigma_i^t = C_{ij}^\infty \bar{\varepsilon}_j^t + C_{ij}^* \sum_{m=1}^N W_m \int_0^t \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau \qquad (7)$$

and then the integral in Eq. (7) is defined as below.

$$q_{jm}^t = \int_0^t \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau \qquad (8)$$

This can be written in two parts for a sufficiently small time increment Δt as follows.

$$q_{jm}^t = \int_0^{t-\Delta t} \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau + \int_{t-\Delta t}^t \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau \qquad (9)$$

The reduced time can also be given by

$$\xi^t = \xi^{t-\Delta t} + \Delta \xi^t \qquad (10)$$

and the increment in the reduced time $\Delta \xi$ is

defined by Eq. (4) as follows.

$$\Delta \xi^t = \int_{t-\Delta t}^{t} \frac{ds}{a_T} \tag{11}$$

Assuming that the degree of cure and the temperature are constant during the sufficiently small time step Δt, Eq. (11) can be approximated as

$$\Delta \xi^t = \frac{\Delta t}{a_T} \tag{12}$$

Using Eq. (10), the first integral term of Eq. (9) is expressed as

$$I_1 = \int_0^{t-\Delta t} \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau$$
$$= \int_0^{t-\Delta t} \exp\left(-\frac{\Delta \xi^t}{\tau_m}\right) \exp\left(-\frac{\xi^{t-\Delta t} - \xi^\tau}{\tau_m}\right)$$
$$\frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau$$

and assuming that the relaxation time τ_m is constant for the sufficiently small time step Δt in order to take advantage of the recursive formula, the above equation can be written approximately as follows (Kennedy and Wang, 1994).

$$I_1 \cong \exp\left(-\frac{\Delta \xi^t}{\tau_m^{t-\Delta t}}\right) \int_0^{t-\Delta t} \exp\left(-\frac{\xi^{t-\Delta t} - \xi^\tau}{\tau_m}\right)$$
$$\frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau$$
$$= \exp\left(-\frac{\Delta \xi^t}{\tau_m^{t-\Delta t}}\right) q_{jm}^{t-\Delta t} \tag{14}$$

If the relaxation time is constant, Eq. (14) will be an exact expression rather than an approximate one. In order to derive the second integral in Eq. (9), let us assume that the change in the strain is linear in an interval of $t - \Delta t < \tau < t$. Namely,

$$\frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} \cong \frac{\bar{\varepsilon}_j^t - \bar{\varepsilon}_j^{t-\Delta t}}{\Delta t} = \text{constant} \tag{15}$$

Using Eqs. (12) and (15), and assuming again that the relaxation time is constant for Δt, the second integral in Eq. (9) is derived as follows.

$$I_2 = \int_{t-\Delta t}^{t} \exp\left(-\frac{\xi^t - \xi^\tau}{\tau_m}\right) \frac{\partial \bar{\varepsilon}_j^\tau}{\partial \tau} d\tau$$
$$\cong \frac{\bar{\varepsilon}_j^t - \bar{\varepsilon}_j^{t-\Delta t}}{\Delta t} \exp\left(-\frac{\xi^t}{\tau_m^t}\right) \int_{t-\Delta t}^{t} \exp\left(\frac{\xi^\tau}{\tau_m^t}\right) d\tau \tag{16}$$
$$= \frac{\bar{\varepsilon}_j^t - \bar{\varepsilon}_j^{t-\Delta t}}{\Delta t} \exp\left(-\frac{\xi^t}{\tau_m^t}\right) \int_{\xi^{t-\Delta t}}^{\xi^t} \exp$$
$$\left(\frac{\xi^\tau}{\tau_m^t}\right) a_T d\xi^\tau$$

Integrating this equation gives

$$I_2 = \frac{\tau_m^t}{\Delta \xi^t} \left[1 - \exp\left(-\frac{\Delta \xi^t}{\tau_m^t}\right)\right] (\bar{\varepsilon}_j^t - \bar{\varepsilon}_j^{t-\Delta t})$$
$$= \Gamma_m^t (\bar{\varepsilon}_j^t - \bar{\varepsilon}_j^{t-\Delta t}) \tag{17}$$

where Γ_m^t is defined as

$$\Gamma_m^t = \frac{\tau_m^t}{\Delta \xi^t} \left[1 - \exp\left(-\frac{\Delta \xi^t}{\tau_m^t}\right)\right] \tag{18}$$

Therefore, substituting Eqs. (14) and (17) into Eq. (9) gives

$$q_{jm}^t = \exp\left(-\frac{\Delta \xi^t}{\tau_m^{t-\Delta t}}\right) q_{jm}^{t-\Delta t} + \Gamma_m^t (\bar{\varepsilon}_j^t - \bar{\varepsilon}_j^{t-\Delta t}) \tag{19}$$

and by substituting Eq. (19) into Eq. (7), the viscoelastic constitutive equation can be formulated as

$$\sigma_i^t = \left[C_{ij}^\infty + C_{ij}^* \sum_{m=1}^{N} W_m \Gamma_m^t\right] \bar{\varepsilon}_j^t \tag{20}$$
$$+ C_{ij}^* \sum_{m=1}^{N} W_m \left[\exp\left(-\frac{\Delta \xi^t}{\tau_m^{t-\Delta t}}\right) q_{jm}^{t-\Delta t}\right.$$
$$\left. - \Gamma_m^t \bar{\varepsilon}_j^{t-\Delta t}\right]$$

where the initial values of Γ_m^o and q_{jm}^o at the time $t = 0$ are 1 and 0, respectively. Equation (20) can be written in the form of vectors as follows.

$$\boldsymbol{\sigma}^t = \boldsymbol{C}^t \bar{\boldsymbol{\varepsilon}}^t + \boldsymbol{H}^t \tag{21}$$

where $\boldsymbol{C}$ represents the first bracket term in Eq. (20) called a time-dependent stiffness matrix and $\boldsymbol{H}$ represents the second term in Eq. (20) called a hereditary stress vector. From a computational point of view, Eq. (20) is much easier to deal with than Eq. (7) because Eq. (20) requires a knowledge of quantities at the previous time $t - \Delta t$, while Eq. (7) requires a knowledge of quantities over the complete history of the response of the material.

3. Formulation of Viscoelastic Finite Element

An equilibrium equation at an arbitrary time t is expressed by the principle of virtual work.

$$\int_V \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV = \int_A \delta \boldsymbol{u}^T \boldsymbol{f}_s dA + \int_v \delta \boldsymbol{u}^T \boldsymbol{f}_b dV \tag{22}$$

where the superscript T denotes transpose, V and A represent volume and area, and $\delta \boldsymbol{u}$, $\boldsymbol{f}_s$, and

f_b represent virtual displacement, surface traction, and body force vectors, respectively. Substitution of Eq. (21) into the above equation leads to

$$\int_V \delta\varepsilon^T C\varepsilon dV = \int_A \delta u^T f_s dA + \int_V \delta u^T f_b dV$$
$$+ \int_V \delta u^T C\varepsilon_{tc} dV - \int_V \delta\varepsilon^T H dV$$
(23)

where ε_{tc} represents thermal and chemical shrinkage strains defined by the second and third terms in the right hand side of Eq. (2).

On the basis of 3-D degenerated shell element and the first order shear deformation theory (Chao and Reddy, 1984 and Panda and Natarajan, 1981), the displacement vector u in an isoparametric element consisting of p nodes is defined as (Bathe, 1982)

$$u = \sum_{k=1}^{P} N^k(\xi, \eta)\, \bar{u}^k + \frac{1}{2}\sum_{k=1}^{P} N^k(\xi, \eta)\, t^k \zeta \Delta V^k$$
(24)

where ξ, η, ζ denote local coordinates of the element and $\bar{u}^k$, N^k, t^k, ΔV^k represent the displacement vector, shape function, thickness, and incremental normal unit vector of the k-th node in the element, respectively. The strain tensor ε is written as

$$\varepsilon = \frac{1}{2}(u\nabla + \nabla u)$$
(25)

Expressing a virtual displacement vector δu and a virtual strain vector $\delta\varepsilon$ by a function of a virtual nodal displacement vector $\delta\bar{u}$ using Eqs. (24) and (25) gives

$$\delta u = N\delta\bar{u}, \quad \delta\varepsilon = B\delta\bar{u}$$
(26)

where N and B represent a shape function matrix and a displacement-strain matrix, respectively (Bathe, 1982). By substituting Eq. (26) into Eq. (23), the viscoelastic finite element formulation at the arbitrary time t is summarized as

$$K\bar{u} = R$$
(27)

where the time-dependent stiffness matrix K and the force vector R are given by

$$K = \int_V B^T CB dV$$
(28)

$$R = \int_A N^T f_s dA + \int_V N^T f_s dA + \int_V B^T C\varepsilon_{tc} dV$$

$$- \int_V B^T H dA$$
(29)

where C is the modulus recalculated to satisfy a condition of plane stress using the modulus C given by Eq. (21) (Bathe, 1982).

4. Degree of Cure, Shift Factor and Relaxation Time

The degree of cure α is defined as the ratio of the heat of reaction released up to time t and the total heat of reaction (Lee et al., 1982 and Dusi et al., 1987). If the degree of cure is zero, it means that composites are uncured. If the degree of cure is one, it means that the composites are fully cured. The degree of cure at the time t is calculated by

$$\alpha(t) = \int_0^t \frac{d\alpha}{dt} dt$$
(30)

Since Hercules AS4/3501-6 composite is used in this paper, the rate of the degree of cure for Hercules 3501-6 resin is determined experimentally as below (Lee et al., 1982).

$$\frac{d\alpha}{dt} = (k_1 + k_2\alpha)(1-\alpha)(0.47-\alpha) \quad (\alpha \le 0.3)$$
(31)

$$\frac{d\alpha}{dt} = k_3(1-\alpha) \quad (\alpha > 0.3)$$

Constants k_1, k_2, k_3 are defined as

$$k_1 = A_1 \exp(-\Delta E_1/RT)$$
$$k_2 = A_2 \exp(-\Delta E_2/RT)$$
$$k_3 = A_3 \exp(-\Delta E_3/RT)$$
(32)

where R is the universal gas constant, and A_1, A_2, A_3 and ΔE_1, ΔE_2, ΔE_3 represent the pre-exponential factors and activation energies of which values are presented in Table 1. The shift factor a_T of AS4/3501-6 composite is functions of the degree of cure and the temperature, and defined as (Kim, 1996)

$$a_T = 10^{\{-a_1\exp\{\frac{1}{\alpha-1}\}-a_2\}(T-T_r)}$$
(33)

where the values of constants a_1 and a_2 are 1.4 and 0.0712, respectively, and T_r denotes the reference temperature determined as 25℃. The relaxation time τ_m is a function of the degree of cure and determined experimentally as follows (Kim,

Table 1 Cure kinetics constants of Hercules 3501-6 resin.

R (J/mol °K)	8.314×10^3
A_1 (min^{-1})	2.101×10^9
A_2 (min^{-1})	-2.014×10^9
A_3 (min^{-1})	1.960×10^5
ΔE_1 (J/mol)	8.07×10^4
ΔE_2 (J/mol)	7.78×10^4
ΔE_3 (J/mol)	5.66×10^4

Table 2 Relaxation time and weighting factors at reference degree of cure. ($\alpha_r = 0.98$)

m	τ_m (min)	W_m
1	2.922137e+1	0.0591334
2	2.921437e+3	0.0661225
3	1.82448e+5	0.0826896
4	1.1031059e+7	0.112314
5	2.8305395e+8	0.154121
6	7.9432822e+9	0.2618288
7	1.953424e+11	0.1835594
8	3.3150756e+12	0.0486939
9	4.9174856e+14	0.0252258

1996).

$$\tau_m(\alpha) = 10^{[\text{Log}\{\tau_m(\alpha_r)\} + f(\alpha) - (\alpha - \alpha_r)\text{Log}(\lambda_m)]}$$
$$f(\alpha) = 0.0536 + 0.0615\alpha + 0.9227\alpha^2 \qquad (34)$$
$$\lambda_m = \frac{10^{9.9}}{\tau_m(\alpha_r)}$$

where α_r represents the reference degree of cure and is 0.98. The relaxation times $\tau_m(\alpha_r)$ corresponding to α_r and the weighting factors W_m are presented in Table 2.

5. Numerical Results and Discussion

Material properties of Hercules AS4/3501-6 composite used in analysis are presented in Table 3. These values were used to calculate the moduli C_{ij} given by Eq. (5). Since stress relaxation in the direction of a fiber can be neglcted, components

Table 3 Mechanical properties of AS4/3501-6 used in calculatins.

Young's modulus, E_{11}	125.38 GPa
Young's modulus, E_{22}	8.1 GPa
Young's modulus, E_{33}	8.1 GPa
Shear modulus, G_{12}	4.1 GPa
Shear modulus, G_{13}	4.1 GPa
Shear modulus, G_{23}	2.75 GPa
Poisson's ratio, ν_{12}	0.25
Poisson's ratio, ν_{13}	0.25
Poisson's ratio, ν_{23}	0.47
Thermal expansion coeff., β_{11}	0.5e-6 1/°C
Thermal expansion coeff., β_{22}	35.3e-6 1/°C
Thermal expansion coeff., β_{33}	35.3e-6 1/°C
Chemical shrinkage coeff., η_{11}	−1.67e-4
Chemical shrinkage coeff., η_{22}	−8.81e-3
Chemical shrinkage coeff., η_{33}	−8.81e-3

C_{11}, C_{12}, and C_{13} related to the fiber direction among the moduli C_{ij} were assumed to be linearly elastic. The fully relaxed modulus C_{ij}^∞ is experimentally determined by $1/7$ times the unrelaxed modulus C_{ij}^u. The element used for the finite element analyses was a 8-node degenerate shell element and $2 \times 2 \times 2$ Gauss integration points were used at each layer of the element to prevent shear locking. Because the accuracy of numerical results depends on the time increment Δt, the finite element analyses were executed for three time increments $\Delta t = 10$ sec, 30 sec, and 1 min in order to examine the convergence of numerical solutions and it was found that there was no big difference between them. Therefore, to save computational time, the time increment $\Delta t = 1$min was used for the numerical analyses presented below.

In order to verify the finite element program developed in this study, the following procedure was carried out. Since there does not exist an exact solution available for comparing with results calculated in this study, we developed a model as following: a laminated plate, of which

both ends were clamped, was considered and the fiber orientation, length, width, and thickness of the plate were 90°, 100mm, 10mm, and 2mm, respectively. If the degree of cure is constant and the temperature is given by $\Delta Th(t)$ where $h(t)$ is a unit step function, the exact solution for a one dimensional problem can be obtained from Eq. (6) as follows.

$$\sigma_2^t = \left[C_{22}^\infty + C_{22}^* \sum_{m=1}^N W_m \exp\left(-\frac{t}{\tau_m a_T} \right) \right] (-\beta_{22} \Delta T)$$
(35)

In the process of deriving the above equation, the total strain ε_2 was given as zero because the ends of the plate were clamped, and σ_2 represents a transverse stress normal to the fiber direction. The plate shown in Fig. 1 was divided by 10 meshes in the length and 2 meshes in the width for the finite element analysis. Total number of elements were 20. When the degree of cure and the temperature change were 0.9 and 170℃, respectively, the results calculated by the exact solution and the finite element analysis were compared in

Fig. 2. The result of the finite element analysis was calculated at the center of the plate. Because two results agree well each other as shown in Fig. 2, validation of the finite element program developed in this paper can be accomplished.

To calculate the residual stresses during cure in this study, a cylindrical laminated shell as shown in Fig. 3 was selected. The radius and length of the shell were 100 mm and the thickness was 2 mm. The arc angle of the shell was 90°. The laminated shell consisted of 8 plies and therefore the thickness of a ply was 0.25 mm. All four edges of the shell were given as free and temperature distribution was assumed to be uniform. For the finite element analysis, the laminated shell was uniformly divided by 4×4 elements. Total numbers of the element and node were 16 and 65, respectively. A cure cycle used in the calculations was presented in Fig. 4. Total cure time was 300 minutes, and temperatures at the first dwell known as a consolidation stage and at the second dwell known as a cure stage were defined as

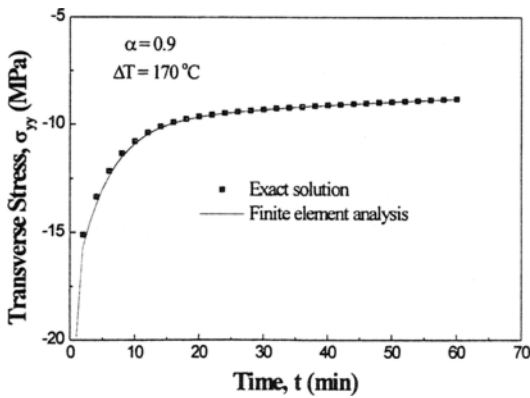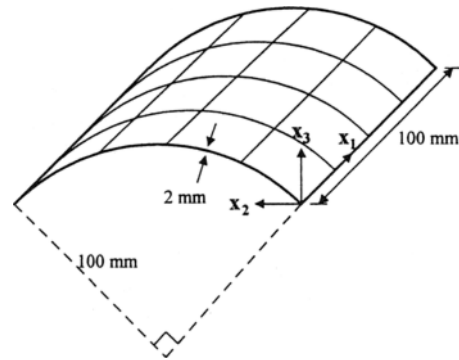

Fig. 1   Geometry of the clamped laminated plate.

Fig. 2   Comparison of the finite element analysis with the exact solution for the transverse stress in the laminated plate.

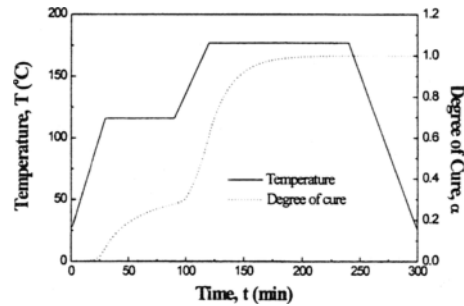Fig. 3   Geometry of the cylindrical laminated shell.

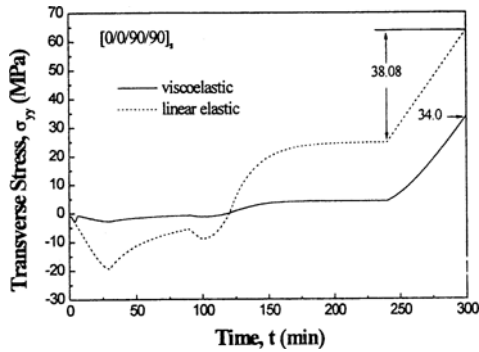Fig. 4   The tempeature and the degree of cure during cure.

Fig. 5 Comparison of the viscoelastic transverse stress with the linear elastic transverse stress during cure.

Fig. 6 Effect of chemical shrinkage on the transverse stress during cure.

Fig. 7 Comparison of longitudinal stresses at the first ply (0 degree) during cure.

116℃ and 177℃, respectively. The degree of cure calculated for this cure cycle was presented in Fig. 4 as dot lines and increased rapidly near the time at 100 minutes.

To investigate the effect of viscoelasticity on the residual stresses in the laminated shell during the cure, the analyses were performed for $[0/0/90/90]_s$ laminate where a positive ply orientation was measured counterclockwise with respect to X_1 axis shown in Fig. 3, and the numerical results were presented in Fig. 5 where transverse normal stresses calculated for both viscoelastic and linear elastic analyses considering thermal deformation and cure-induced chemical shrinkage simultaneously were compared. These stresses were calculated at the first ply 0° and near the center of the shell, and quite different as shown in Fig. 5. Especially, it was found that the viscoelastic stress was considerably relaxed with the advance in cure, and that the final residual stress for the viscoelastic analysis was much lower than that for the linear elastic analysis. Generally, calculating the residual stress using the linear elastic analysis, the stress is assumed to be free at the second dwell called the cure stage and calculated only at the cooling stage. Under this assumption, the calculated linear elastic stress was 38.08 MPa during cooling as shown in Fig. 5 and the final viscoelastic stress was 34.0 MPa at the end of cure. This indicates 12% difference with respect to the viscoelastic stress and, in addition, such a linear elastic analysis has disadvantage that can not calculate the residual stress in the total process of
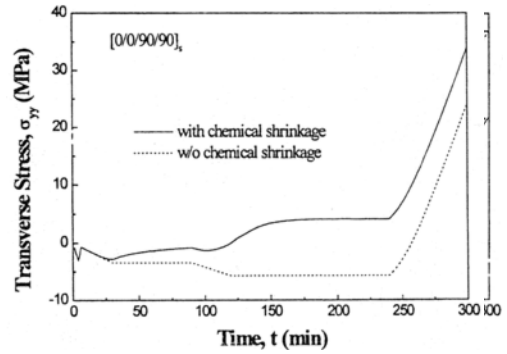
cure. Next, the result of the analysis performed for examining the effect of chemical shrinkage on the residual stress was shown in Fig. 6. As shown in the figure, the final residual stress calculated with the chemical shrinkage was greater than that calculated without the chemical shrinkage. Therefore, it was recognized that the chemical shrinkage caused the final residual stress to be increased.

Finally, the analysis was executed for laminated shells consisting of $[0/0/90/90]_s$, $[0/45/90/-45]_s$, $[0/0/45/-45]_s$, $[0/60/45/-30]_s$ stacking sequences. Longitudinal and transverse normal stresses are presented in Fig. 7 and Fig. 8. Each stress was calculated at the first ply 0° and near the center of the shell. For cross-ply and quasi-isotropic laminates, the transverse stresses as well as the longitudinal stresses were almost the same each other. Especially, the transverse normal stresses for all stacking sequences show to be
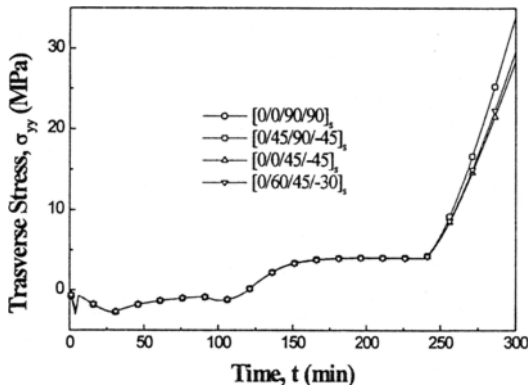
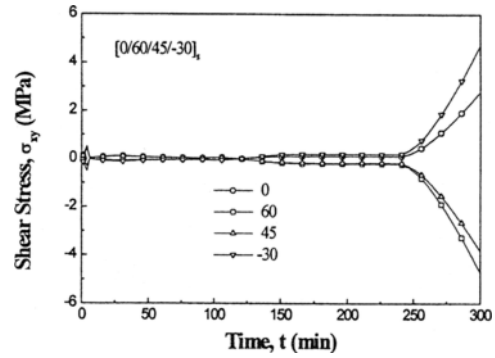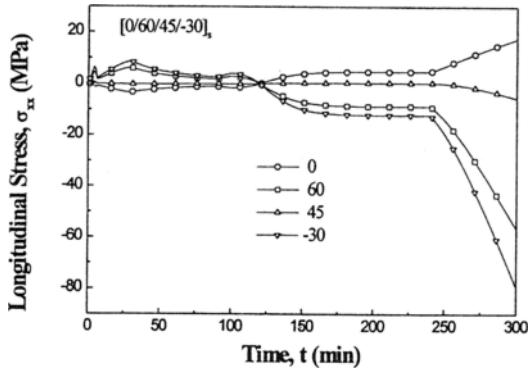**Fig. 8** Comparison of transverse stresses at the first ply (0 degree) during cure.

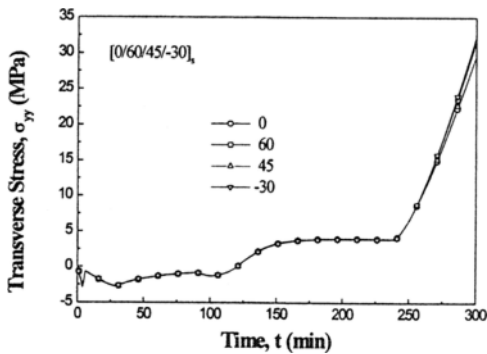**Fig. 9** Longitudinal stress development at each ply during cure.

Fig. 10 Transverse stress development at each ply during cure.

nearly identical before the cooling stage. Residual stresses at each ply of $[0/60/45/-30]_s$ laminated shell were presented in Figs. 9–11. Final maximum longitudinal, transverse, and shear stresses were occurred at $-30°$ ply, and transverse stresses were also almost the same irrespective of ply

Fig. 11 Shear stress development at each ply during cure.

orientation before the cooling stage

5. Conclusions

In this study, the viscoelastic finite element analysis was performed to investigate the residual stresses occurred in laminated shells during the cure. The viscoelastic constitutive equation that can describe the stress relaxation during the cure was defined as functions of degree of cure and temperature, and derived as a recursive formula used conveniently for numerical analyses. The finite element program was developed on the basis of 3-D degenerated shell element and the first order shear deformation theory, and verified by comparing with the exact solution of the one dimensional problem.

To investigate the effect of viscoelasticity on the residual stresses in the laminated shell during the cure, both the viscoelastic and linear elastic analyses were performed for $[0/0/90/90]_s$ laminate considering thermal deformation and chemical shrinkage simultaneously. The results showed that there were big differences between the viscoelastic stresses and the linear elastic stresses. The effect of the chemical shrinkage and various stacking sequences on the residual stresses in the laminated shells during the cure was also examined.

In conclusion, to predict more accurately the residual stresses occurred in the laminated shell during the cure, the viscoelastic analysis considering the stress relaxation and the chemical shrinkage as well as the thermal deformation should be

carried out.

References

Bathe, K. J., 1982, *Finite Element Procedures in Engineering Analysis*, Prentice Hall.

Bogetti, T. A. and Gillespie, Jr. J. W., 1992, "Process-induced Stress and Deformation in Thick-Section Thermoset Composite Laminate," *Journal of Composite Materials*, Vol. 26, pp. 626 ~660

Chao, W. C. and Reddy, J. N., 1984, "Analysis of Laminated Composite Shells Using a Degenerated 3-D Element," *International Journal for Numerical Methods in Engineering*, Vol. 20, pp. 1991~2007.

Dusi, M. R., Lee, W. I., Ciriscioli, P. R. and Springer, G. S., 1987, "Cure Kinetics and Viscosity of Fiberite 976 Resin," *Journal of Composite Materials*, Vol. 21, pp. 243~261.

Flaggs, D. L. and Crossman, F. W., 1981, "Analysis of the Viscoelastic Response of Composite Laminates During Hygrothermal Exposure," *Journal of Composite Materials*, Vol. 15, pp. 21~40.

Hahn, H. T., 1976, "Residual Stresses in Polymer Matrix Composites Laminates," *Journal of Composite Materials*, Vol. 10, pp. 266~277.

Hahn, H. T. and Pagano, N. J., 1975, "Curing Stress in Composite Laminates," *Journal of Composite Materials*, Vol. 9, pp. 91~105.

Henriksen, M., 1984, "Nonlinear Viscoelastic Stress Analysis-A Finite Element Approach," *Computers & Structures*, Vol. 18, pp. 133~139.

Hodges, J., Yates, B., Darley, M. I., Wostenholm, G. H., Clement, J. F. and Keates, T. F., 1989, "Residual Stresses and the Optimum Cure Cycle for an Epoxy Resin," *Journal of Material*

Science, Vol. 24, pp. 1984~1990.

Kennedy, T. C. and Wang, M., 1994, "Three-Dimensional Nonlinear Viscoelastic Analysis of Laminated Composties," *Journal of Composite Materials*, Vol. 28, pp. 121~132.

Kim, Y. K., 1996, "Process-Induced Viscoelastic Residual Stress Analysis of Graphite-Epoxy Composite Structures," Ph. D. Dissertation, Department of Aeronautical and Astronautical Engineering, University of Illinois at Urbana-Champaign.

Kim, K. S. and Hahn, H. T., 1989, "Residual Stress Development During Processing of Graphite/Epoxy Composites," *Composites Science and Technology*, Vol. 36, pp. 121~132.

Lee. W. I., Loos, A. C. and Springer, G. S., 1982, "Heat of Reaction, Degree of cure, and Viscosity of Hercules 3501-6 Resin," *Journal of Composite Materials*, Vol. 16, pp. 510-520.

Lin, K. Y. and Hwang, I. H., 1989, "Thermo-Viscoelastic Analysis of Composite Materials," *Journal of Composite Materials*, Vol. 23, pp. 554 ~569.

Panda, S. C. and Natarajan, R., 1981, "Analysis of Laminated Composite Shell Structures by Finite Element Method," *Computers & Structures*, Vol. 14, pp. 225~230.

White, S. R. and Hahn, H. T., 1992, "Process Modelling of Composite Materials: Residual Stress Development during Cure. Part I. Model Formulation," *Journal of Composite Materials*, Vol. 26, pp. 2402~2422.

White, S. R. and Hahn, H. T., 1992, "Process Modelling of Composite Materials: Residual Stress Development during Cure. Part II. Experimental Validation," *Journal of Composite Materials*, Vol. 26, pp. 2423~2453.